Pergamon

Mechanics Research Communications, Vol. 28, No. 5, pp. 525–534, 2001

0093-6413/01/$–see front matter

PII: S0093-6413(01)00205-1

Optimal Control of the Dynamic Response of an Anisotropic Plate with Various Boundary Conditions

Y.G. Youssif, M.E. Fares and M.A. Hafiz

Department of mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt.

(Received 22 November 2000; accepted for print 28 August 2001)

Abstract: The problem of minimising the dynamic response of an anisotropic rectangular plate with minimum possible expenditure of force is presented for various cases of boundary conditions. The plate has a principal direction of anisotropy rotated at an arbitrary angle relative to the coordinate axes. This orientation angle has been taken as an optimisation design parameter. The control problem is formulated as an optimisation problem by using a performance index, which comprises a weight sum of the control objective and penalty function of the control force. The explicit solutions for the closed-loop distributed control function is obtained by means of Liapunov-Bellman theory. To assess the present solution, numerical results are presented to illustrate the effect of anisotropy ratio, orientation angle, aspect ratio and boundary conditions on the control process.

Keywords: Optimal control, Minimisation of dynamic response, Anisotropic plates.

1. Introduction

The rapid development of various industrial fields requires new materials that can serve useful functions under certain conditions. In aerospace industry and many other engineering applications, the suppression of excessive vibrations occurring in large structures represents one of the most pressing and difficult problems facing structural designers. An effective means of suppressing excessive vibrations is by active structural control. Thus, there is need for new light materials possessing a high degree of flexibility and with very low natural damping. These factors motivated the development of more accurate tools of analysis and rigorous design methods. Therefore, the optimal control problems of dynamical systems have long been a main subject of many studies and up-to-date lists of publications in this area is given in survey articles [1-5].

Most recently, the strong interaction between structural control and design optimisation has been recognized. As a result, simultaneous design and control has been the subject of several research studies with a view towards integrating optimal design and active control in a single formulation. For instance, in Refs. [6-11], the design control problem was formulated as a constrained optimisation problem.

A series of publications has been concerned with the fundamental considerations of these approaches and their applications to different dynamical systems. Sloss and others [12,13] presented a maximum principle for the optimal control of a general class of dynamical systems with distributed parameters. Within the theoretical framework of these studies, optimal distributed control results were obtained for membranes by Sadek and Adali [14], for thick beams by Sadek and others [15-17], for continuous beams by Sadek et al. [18], for Mindlin-Timoshenko plates by

Sadek et. al. [19] and for orthotropic plates by Adali et al. [20]. Other studies may be found in [21-26]. For these studies, however, there have been considerably few papers concerned with anisotropic structures with various boundary conditions.

The objective of the current work deals with the optimal control of the dynamic response of an anisotropic rectangular plate possessing a principal direction of anisotropy rotated at an arbitrary angle relative to the coordinate axes. This orientation angle may be taken as optimisation design parameter. Various cases of boundary conditions are considered. The present control problem is the minimisation of the dynamic response of a damped plate with the minimum possible expenditure of force. Control over the plate is exercised by distributed forces, which translate into force in the actual implementation of the control mechanism. The dynamic response of the anisotropic plate comprise its deflection and velocity which constitute multiple objectives of the control problem together with the expenditure of force. The dynamic response is related to the energy of the structure, which is subject to initial disturbances. A quadratic functional of the dynamic response is specified as the control performance index. The expenditure of force is limited by attaching a functional of force to the objective functional as a penalty term. The necessary and sufficient conditions for optimal stabilization in Liapunov-Bellman sense [27] are used to determine the control force and deflections. Numerical example is given to study influences of anisotropy ratio, orientation angle, aspect ratio, and boundary conditions on control process.

2. Formulation of the problem

Consider an anisotropic rectangular plate of length a, width b, and thickness h .The mid-plane of the plate coincides with xy- plane and normal to z- axis as shown in figure 1.

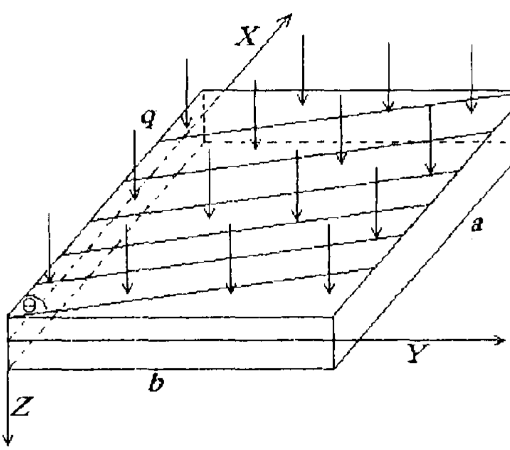

Figure 1. The plate with fibers orientated at an arbitrary angle θ relative to the coordinate system.

The material of the plate is assumed to possess a principal direction of elasticity rotated at an angle θ relative to x- direction. Let the plate be subjected to distributed force $q(x, y, t)$ act on the upper surface of the plate.

The fundamental differential equation governing the motion of the plate is given by [28]:

$$\rho h\ddot{w} + D_{11}^{*}w_{,xxxx} + 2D_{12}^{*}w_{,xxyy} + D_{22}^{*}w_{,yyyy} + 4D_{66}^{*}w_{,xxyy} + 4\left(D_{16}^{*}w_{,xxxy} + D_{26}^{*}w_{,xyyy}\right) = q \ , \tag{1}$$

where w is the plate deflection in the z- direction, ρ is the material density, the superposed dot denotes differentiation with respect to time and $(\ )_{,\zeta}$ denotes partial differentiation with respect to corresponding coordinate, $D_{ij}^{*} = B_{ij}^{*}h^3/12$ are the rigidities of the plate, which are related to the fundamental elastic constants B_{ij} are presented in [28]:

The present control problem accounts for various cases of boundary conditions at edges, i.e., when the plate edges are Simply Supported (S), or Clamped (C) or Free (F), or when mixed of these boundary conditions are prescribed over edges. These boundary conditions on edges perpendicular to x-axis (for example):

$$C: \ w = w_{,x} = 0,$$

$$S: \ w = D_{11}^{*}w_{,xx} + D_{12}^{*}w_{,yy} - 2D_{16}^{*}w_{,xy} = 0,$$

$$F: \ D_{11}^{*}w_{,xx} + D_{12}^{*}w_{,yy} + 2D_{16}^{*}w_{,xy} = D_{11}^{*}w_{,xxx} + 3D_{16}^{*}w_{,xxy} + \left(D_{12}^{*} + 2D_{66}^{*}\right)w_{,xyy} + D_{26}^{*}w_{,xyy} = 0. \tag{2}$$

Also, we assume that the plate is subjected to the following initial conditions:

$$w(x,y,0) = \psi(x,y), \qquad \dot{w}(x,y,0) = \phi(x,y). \tag{3}$$

3. Optimal control problem

The objectives of the present study are to determine the optimal control force q^{o} and optimal design variable θ_{opt} to minimise the dynamic response of the lamina in a specified time $0 \le t \le \tau \le \infty$. the dynamic response of the plate is measured by a cost functional related to the energy of the system. The strain and kinetic energies of the plate made up of linear elastic material, respectively are [28] :

$$J_1(q,\theta) = \frac{1}{2}\int_0^{\infty}\int_0^{a}\int_0^{b}\left[D_{11}^{*}w_{,xx}^2 + 2D_{12}^{*}w_{,xx}w_{,yy} + D_{22}^{*}w_{,yy}^2 + 4D_{66}^{*}w_{,xy}^2 + 4\left(D_{16}^{*}w_{,xx} + D_{26}^{*}w_{,yy}\right)w_{,xy}\right]dx\,dy\,dt \ , \tag{4}$$

$$J_2(q,\theta) = \frac{1}{2}\rho h\int_0^{\infty}\int_0^{a}\int_0^{b}\dot{w}^2 dxdydt \tag{5}$$

The mathematical formulation of the cost functional may be chosen as:

$$J(q,\ \theta) = \xi_1 J_1 + \xi_2 J_2 + \xi_3 J_3, \tag{6}$$

where $\xi_i > 0$, $(i = 1, 2, 3)$ are constant weighting factors and the functional J_3 is a penalty term involving the control function $q \in L^2$, where L^2 denotes the set of all bounded square integrable functions on $\{0 \leq x \leq a, 0 \leq y \leq b, 0 \leq t \leq \tau \leq \infty\}$, and given by:

$$J_3(q,\theta) = \int_0^\tau \int_0^b \int_0^a q^2(x,y,t)\, dx dy, \tag{7}$$

Thus, the dynamic response of the plate is expressed as functionals of w, its spatial derivatives and $\dot{w}$ given by J_1 and. J_2 Then, the present multiobjective control problem is to determine: firstly, the optimal control function q^o from the minimisation condition of the functional J and secondly, the optimal orientation angle θ_{opt} which minimising the total elastic energy J_{12} $(=J_1+J_2)$.

4. Solution procedure

Under the above specific conditions, we can expand the displacement function w and the control function q in the following double series:

$$w = \sum_{m,n} W_{mn}(t) X(x) Y(y), \qquad q = \sum_{m,n} Q_{mn}(t) X(x) Y(y), \tag{8}$$

where W_{mn} and Q_{mn} are unknown functions of time, X (x) and Y (y) are continuous orthonormed eigenfunctions which satisfy the boundary conditions given in (2) and represent approximate shape of the deflected surface of the free vibrating plate. These functions for the different cases of boundary conditions take the following forms [29]:

$SS: X(x) = \sin \mu_m x, \qquad \mu_m = m\pi / a.$

$CC: X(x) = \sin \mu_m x - \sinh \mu_m x - \eta_m (\cos \mu_m x - \cosh \mu_m x),$

$\eta_m = (\sin \mu_m a - \sinh \mu_m a)/(\cos \mu_m a - \cosh \mu_m a), \quad \mu_m = (m + 0.5)\pi / a.$

$CS: X(x) = \sin \mu_m x - \sinh \mu_m x - \eta_m (\cos \mu_m x - \cosh \mu_m x),$

$\eta_m = (\sin \mu_m a + \sinh \mu_m a)/(\cos \mu_m a + \cosh \mu_m a), \quad \mu_m = (m + 0.25)\pi / a.$

$CF: X(x) = \sin \mu_m x - \sinh \mu_m x - \eta_m (\cos \mu_m x - \cosh \mu_m x),$

$\eta_m = (\sin \mu_m a + \sinh \mu_m a)/(\cos \mu_m a + \cosh \mu_m a), \quad \mu_1 = 1.875/a, \quad \mu_2 = 4.694/a,$

$\mu_3 = 7.855/a, \quad \mu_4 = 10.996/a$ and $\mu_m = (m - 0.25)\pi / a$ for $m \geq 5$.

Substituting formulae (8) into equation (1), then, multiplying both sides of the resulting equation by $X(x)Y(y)$, and integration over the domain of solution, we get:

$$\ddot{W}_{mn} + \omega_{mn}^2 W_{mn} = \frac{1}{h\rho} Q_{mn}, \tag{9}$$

$$\omega_{mn}^2 = \frac{1}{hI_6\rho}\left[D_{11}^{*}I_1 + 2D_{16}^{*}I_2 + 2\left(D_{12}^{*} + 2D_{66}^{*}\right)I_3 + 4D_{26}^{*}I_4 + D_{22}^{*}I_5\right], \tag{10}$$

Substituting the relations (8) into expressions (4) and (5), we can easily get:

$$J_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\int_0^{\infty} e_1 W_{mn}^2 dt, , \qquad J_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\int_0^{\infty} e_2 \dot{W}_{mn}^2 dt, \tag{11}$$

$$e_1 = \tfrac{1}{2}\xi_1\left(D_{11}^{*}I_7 + 2D_{12}^{*}I_8 + 2D_{22}^{*}I_9 + 4D_{66}^{*}I_{10} + 4D_{16}^{*}I_{11} + 4D_{26}^{*}I_{12}\right), \quad e_2 = \tfrac{1}{2}h\rho\xi_2 I_6 ,$$

$$\left(I_1, I_2, I_3, I_4, I_5, I_6\right) = \int_0^a\int_0^b\left(X_{,xxxx}Y, X_{,xxx}Y_{,y}, X_{,xx}Y_{,yy}, X_{,x}Y_{,yyy}, XY_{,yyyy}, XY\right)XYdxdy \ ,$$

$$\left(I_7, I_8, I_9, I_{10}, I_{11}, I_{12}\right) = \int_0^a\int_0^b\left(X_{,xx}^2Y^2, X_{,xx}Y_{,yy}, X^2Y_{,yy}^2, X_{,x}^2Y_{,y}^2, X_{,xx}X_{,x}Y_{,y}Y, X_{,x}X\ Y_{,yy}Y_{,y}\right)dxdy.$$

Using the expressions (7) and (11) in (6), the functional J takes the form:

$$J = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} J_{mn}, \tag{12}$$

$$J_{mn} = \int_0^{\infty}\left(e_1W_{mn}^2 + e_2\dot{W}_{mn}^2 + e_3Q_{mn}^2\right)dt, \quad e_3 = \xi_3 I_6 . \tag{13}$$

To minimise the functional J, we apply Liapunov-Bellman theory [27] that gives the minimisation condition in the form:

$$\min_{Q=Q^o}\left[\frac{\partial V_{mn}}{\partial W_{mn}}\dot{W}_{mn} + \frac{\partial V_{mn}}{\partial \dot{W}_{mn}}\ddot{W}_{mn} + \bar{J}_{mn}\right] = 0 , \tag{14}$$

where V_{mn} is a Liapunov function and may be chosen in the form:

$$V_{mn} = \varphi_{mn}W_{mn}^2 + 2\varepsilon_{mn}W_{mn}\dot{W}_{mn} + \eta_{mn}\dot{W}_{mn}^2, \tag{15}$$

$\bar{J}_{mn}$ is the integrand of (13), φ_{mn}, ε_{mn} and η_{mn} are parameters chosen according to the condition that the Liapunov function V_{mn} is positive definite. Then, from expressions (13)-(15), we can obtain the optimal control force in the form:

$$Q_{mn}^o = \frac{-1}{\rho h e_3}\left(\varepsilon_{mn}W_{mn} + \eta_{mn}\dot{W}_{mn}\right), \tag{16}$$

substituting equations (9) and (16) into (14), and equating the coefficients of W_{mn}^2, $\dot{W}_{mn}^2$ and $W_{mn}\dot{W}_{mn}$ by zero, we get a system of equations, the general solution of this system is:

$$\varepsilon_{mn} = -e_4\left(\omega_{mn}^2 - \sqrt{\omega_{mn}^4 + e_1/e_4}\right), \qquad \eta_{mn} = \sqrt{\left(2\varepsilon_{mn} + e_2\right)\ e_4} ,$$

$$\varphi_{mn} = \varepsilon_{mn}\left(\omega_{mn}^2 + \eta_{mn}/e_4\right), \qquad e_4 = \rho^2h^2e_3. \tag{17}$$

The signs before the square roots are chosen according to the condition on Liapunov functions. On the other hand, we can rewrite the equation (9) as follows:

$$\ddot{W}_{mn}+\alpha_{mn}\dot{W}_{mn}+\Omega_{mn}^{2}W_{mn}=0\,,\qquad \alpha_{mn}=\frac{\eta_{mn}}{h^{2}\rho^{2}e_{3}}\,,\qquad \Omega_{mn}^{2}=\omega_{mn}^{2}+\frac{\varepsilon_{mn}}{h^{2}\rho^{2}e_{3}}\,. \tag{18}$$

The solution of equation (18) for which $2\Omega_{mn}>\alpha_{mn}$, is given by:

$$W_{mn}^{o}=e^{-\alpha_{mn}t/2}\left[\beta_{mn}\cos(\nu_{mn}t)+\gamma_{mn}\sin(\nu_{mn}t)\right],\qquad \nu_{mn}^{2}=\Omega_{mn}^{2}-\tfrac{1}{4}\alpha_{mn}^{2}\,, \tag{19}$$

where β_{mn} and γ_{mn} are unknown coefficients which may be obtained from the initial conditions (3) by expanding it in series, then:

$$\gamma_{mn}=\frac{\alpha_{mn}\beta_{mn}+2A_{mn}}{2\nu},\qquad (\beta_{mn},A_{mn})=\frac{4}{ab\omega^{2}}\int_{0}^{a}\int_{0}^{b}(\psi,\phi)XY\,dx\,dy. \tag{20}$$

From the above expressions we can obtain the optimal control force and the total elastic energy for various boundary conditions.

5. Numerical results and discussion

In this section, numerical results for the optimal deflections w^{o}, force q^{o} and the total elastic energy J_{12} are presented when the plate is orthotropic, In this case, the engineering constants are introduced instead of the elastic constants from the relations:

$$B_{11}=\frac{E_{1}}{1-\nu_{12}\nu_{21}},\quad B_{12}=\frac{\nu_{12}E_{2}}{1-\nu_{12}\nu_{21}},\quad B_{22}=\frac{E_{2}}{1-\nu_{12}\nu_{21}},\quad B_{66}=G_{12}\,,\quad B_{16}=B_{26}=0, \tag{21}$$

where E_i are Young's moduli; ν_{ij} are Poisson's ratios and G_{ij} are shear moduli. The Poisson's ratios and Young's moduli are related by the reciprocal relations $\nu_{ij}E_{j}=\nu_{ji}E_{i}\ ,(i,\ j=1,\ 2)$. The initial conditions (3) are chosen in the form :

$$w\,(x,y,0)=0.2X(x)\,Y(y),\qquad \dot{w}\,(x,y,0)=0. \tag{22}$$

We introduce the dimensionless quantities $\bar{x}=x/a,\ \bar{y}=y/b$,

$$\bar{w}=w/a,\quad \bar{w}^{o}=w^{o}/a\,,\quad \bar{q}=a^{3}q/(E_{2}h^{3})\,,\quad \bar{q}^{0}=a^{3}q^{o}/(E_{2}h^{3}),\quad T=\frac{ht}{a^{2}}\sqrt{E_{2}/\rho}. \tag{23}$$

In all calculations, unless otherwise stated, the following parameters are used ,

$$E_{1}/E_{2}=40,\quad G_{12}/E_{2}=0.5,\quad \nu_{12}=0.25,\quad \xi_{1}=\xi_{2}=1,\quad \xi_{3}=0.1,$$

which are typical of carbon fiber reinforced plastic. All curves for displacement and force functions are given at the midpoint $\bar{x}=0.5,\ \bar{y}=0.5$, and the four letters of the boundary conditions (*SSCC*, *SCCF*, ... ,etc.) with its order from left to right indicate the kind of fixing at the plate edges $x=0,\ x=a,\ y=0$ and $y=b$, respectively.

Table 1 gives values of the optimal orientation angle θ_{opt} by degrees, at which the total elastic energy J_{12} takes minimum values for different cases of boundary conditions and for different values of orthotropy ratio E_1/E_2 and aspect ratio a/b. Note that, the orientation angle θ which gives the minimum energy strongly depends on the material and geometric parameters of the plate and on the boundary conditions. Fig. 2 shows curves of elastic energy plotted against the orthotropy ratio E_1/E_2 for the cases *CSCC* and *CCCF*. Observe that, the control process using the orientation angle θ considerably reduces the elastic energy of the plate as compared to non-optimal one. Moreover, the advantage of this type of control becomes more obvious in plates with high orthotropy ratio E_1/E_2.

Fig. 3 shows elastic energy curves plotted against the aspect ratio a/b for the cases *SSCC* and *SSSS*. These curves indicate that in short plates, $a/b < 2$, the effect of the orientation angle θ on the optimisation control process plays more significant role in minimising the dynamic response of the plate. Fig. 4 contains curves of the elastic energy J_{12} against the time and aspect ratio a/b. The behavior of the damped deflection of the plate with respect to time is studied in Fig. 5, for cases *CCCC* and *CFCF*. Figs 4 and 5 confirm the previous discussion for all cases of boundary conditions.

Figs 6 and 7 show behavior of the dimensionless optimal control force $\bar{q}^o$ with respect to time. These figures confirm that the control by the orientation angle θ not only plays an efficient role in minimising the dynamic response, but it also contributes significantly in reducing the expenditure of the used force for all cases of boundary conditions. Thus, the present simultaneous control using design parameters and a distributed force is considered an effective mean for damping the dynamic response.

a/b	E_1/E_2	*SSSS*	*SSCS*	*SSCC*	*CSCC*	*CSCS*	*CCCC*	*CCCF*	*SSCF*	*CFCF*	*CSCF*
	2	25.6	45	62	39.7	18.9	0	0	0	29	8.1
0.8	10	35	45	54	42.3	31.7	0	0	0	20	8.7
	40	35.5	45	53	42.6	32.5	0	0	0	19	8.9
	2	45	62.4	90	90	45	45	0	0	55	8.6
1	10	45	54.8	90	64.4	45	45	0	0	68	9.2
	40	45	54.4	90	62.9	45	45	0	0	70	9.5
	2	63.5	90	90	90	71.1	90	0	0	63.5	8.8
1.25	10	55	84.6	90	90	58.3	90	0	0	69.5	9.3
	40	54.5	80.1	90	90	67.5	90	0	0	70	9.7
	2	90	90	90	90	90	90	0	0	66.5	8.9
1.5	10	76	90	90	90	90	90	0	0	70	9.1
	40	74	90	90	90	90	90	0	0	70	9.5
	2	90	90	90	90	90	90	0	90	70	9.5
2	10	90	90	90	90	90	90	0	90	71	8.3
	40	90	90	90	90	90	90	0	90	70.5	8.6

Table 1. Values of optimal orientation angle θ_{opt} which minimising total elastic energy against E_1/E_2 and a/b.

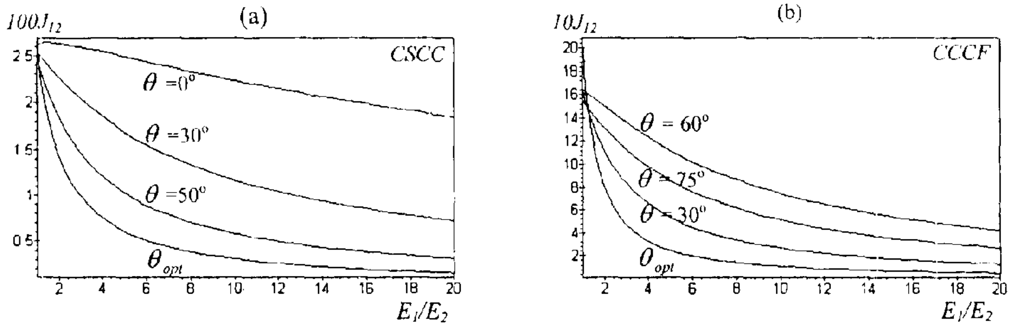

Fig.2. Elastic energy plotted against E_1/E_2, a/b =2 for:
(a) *CSCC* case; (b) CCCF case.

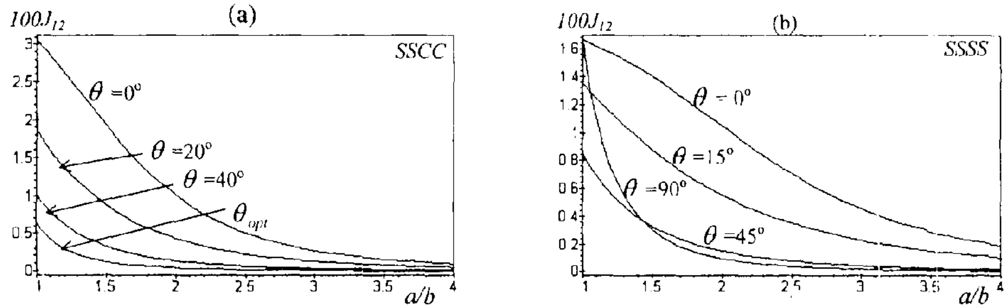

Fig.3. Elastic energy plotted against aspect ratio a/b, for various values of θ for:
(a) *SSCC* case; (b) *SSSS* case.

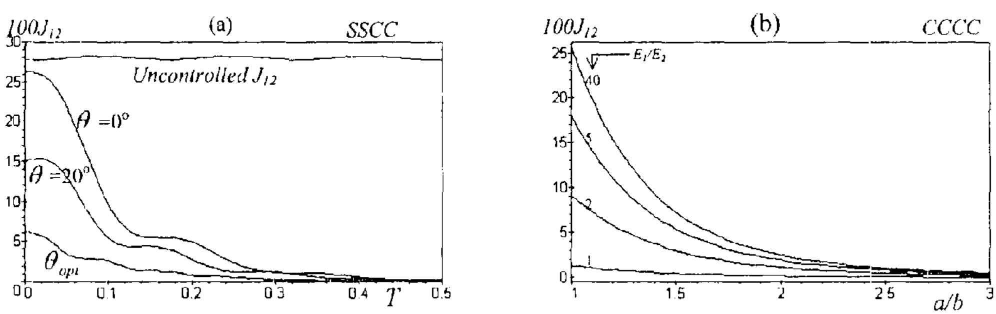

Fig.4. (a) Elastic energy plotted against time T, a/b =1 for *SSCC* and for various values of θ .
(b) Elastic energy plotted against a/b for various values of E_1/E_2 and *CCCC*.

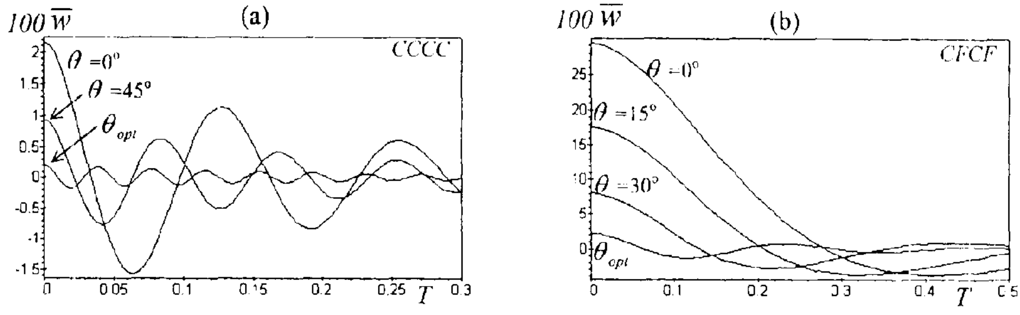

Fig.5. The deflection at the center of the plate plotted against time, $a/b=2$ for:
(a) *CCCC*, case; (b) *CFCF* case.

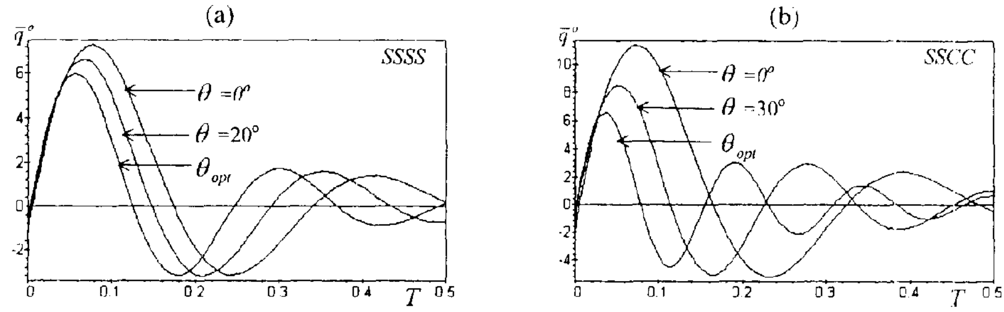

Fig.6. The optimal control force $\overline{q}^o$ plotted against time, $a/b=1$ for:
(a) *SSSS* case; (b) *SSCC* case.

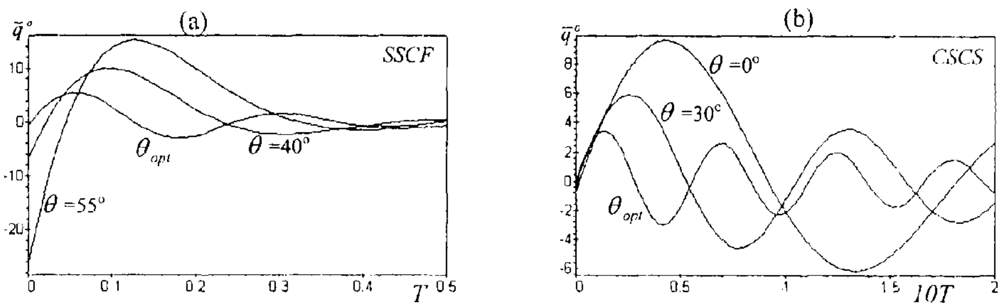

Fig.7. The optimal control force $\overline{q}^o$ plotted against time, $a/b=2$ for:
(a) *SSCF* case; (b) *CSCS* case.

References:

1. J. N. Yang and T.T. Soong, Recent advances in active control of civil engineering structures.*Probabilist. Engng Mech.* 3, 179-188 (1988).
2. R. K. Miller, S. F. Masri, T. J. Dehganyer and T. K. Caughey, Active vibration control of large civil structures. *ASCE J. Engng Mech.* 114, 1542-1570 (1988).
3. A. M. Reinhorn and G. D. Manolis, Recent advances in structural control . *Shock Vibr. Dig.* 21, 3-8 (1989).
4 E. Arthur Jr. Bryson, Optimal control- 1950 to 1985. *IEEE Control systems.*26-33 (1996).
5. J. Hector Sussmann and C.Willems, 300 years of optimal control: from the brachystochrone to the maximum principle. *IEEE Control systems*. 32-44 (1997).
6. M.A. Langthjem,Y. Sugiyama, (2000). Optimum design of cantilevered columns under the combined of conservative and nonconservative loads Part II: The damped case. Computers& Structures 74,399-408
7. Q. Wang, K.M. Liew, A Theory for reduced order control design of plate systems. *Journal of Applied Mechanics* 64, 532-537 (1997).
8. R. V. Grandhi, Structural and control optimization of space structures. *Struct.* 31, 139-150 (1989).
9. N. S. Khot, Structure/control optimization to improve the dynamic response of space structures.*Comput. Mech.* 3, 179-186 (1988).
10. V. B. Venkayya and V. A. Tischler, Frequency control and its effect on the dynamic response of flexible structures. *AIAA Jnl* 23, 1768-1774 (1985).
11. V. Komokov, Simultanrous control and optimization for elastic system. *Workshop on Applications of Distributed System Theory to the Control of Large Space Structures,* 14 -16 July 1982, Jet Propulsion Laboratory, NASA (Edited by G. Rodriguez), pp.391-408 (1983).
12. J. M. Sloss, J. C. Bruch, Jr. and I. S. Sadek, A maximum principal for non-conservative self-adjoint systems. *IMA J. Math. Control & Inf.* 6, 199-216 (1989).
13. U. Ledzewicz, Extension of the local maximum principle control problem. *Journal of Optimization Theory and Applications*,77,No.3, 661-680 (1993).
14. I. S. Sadek and S. Adali, Control of the dynamic response of a damped membrane by distributed forces. *J. Sound Vibration* 96, 391-406, (1984).
15. I. S. Sadek, Variational methods for the distributed control of a vibrating beam. *Optimal Control Appl. Methods.* 9, 79-85, (1988).
16. Julio F. Davalos, Pizhong Qiao, A computational approach for analysis and optimal design of FRP beams. *Computers & Strutures*,70, 169-183 (1999).
17. Seung-Yop Lee,C.D. Mote,Jr. Wave characteristics and vibration control of translating beams by optimal boundary damping. *Journal of vibration and acoustics*,121, 18-25 (1999).
18. I.S.sadek, S. Adali, J. M. Sloss and J. C. Bruch, Jr., Optimal distributed control of a continuous beam with damping. *J. Sound Vibration* 324, 207-218, (1987a).
19. I. S. Sadek, J. M. Sloss, J. C. Bruch, Jr. and S. Adali. Structural control to minimize the dynamic response of Mindlin-Timoshenko plates. *J. Franklin Inst.* 324, 97-112, (1987b).
20. S. Adali, I. S. Sadek, J. M. Sloss and J. C. Bruch, Jr. Distributed control of layered orthotropic plates with damping. *Optimal Control Appl. Methods* 9, 1-17, (1988).
21. Y.G.Youssif, M.N.M. Allam and A.E. Alamir. Optimal stabilization of a compressed elastic shallow shell. *J.Egypt. Math. Soc.* 6(2), 233-244, (1998).
22. Y.G.Youssif. The influence of the in-plane compression on the optimal stability of a vibrating cylindrical shell. *J. Egypt. Math. Soc.* 7(2) (1999).
23. N. Tanaka, Y. Kikushima, Optimal vibration feedback control of an Euler-Bernoulli beam: Toward realizationof the active Sink method. *Journal of vibration and acoustics*,121, 174-182 (1999).
24. Dong-Hua Shi, S.H.Hou and De-Xing Feng, Feedback stabilization of a Timoshenko beam with an end mass. *J. Sound Vibration* 69(2), 285-300, (1998).
25 Y.Xiang, C.M. Wang, and S. Kitipornchai, Optimal design of internal ring support for rectangular plates against vibration or buckling. *J. Sound Vib.* 116, 545-554 (1996).
26 J.C. Bruch, S. Adali, J.M. Sloss and I.S. Sadek, Optimal design and control of cross-ply laminate for maximum frequency and minimum dynamic response. Computers & structures. 37, 87-94 (1990).
27. M. S. Gabralyan. The stabilization of mechanical systems under continuous forces. *YGU Yervan* 2, 47-56, (1975).
28. S. G. Lekhnitskii, *Theory of Elasticity of anisotropic plates*, Gordon and Breach, New York, (1968).
29. J. N. Reddy. Mechanics of Composite Materials and structures. Theory and Analysis. CRC Press. Florida. 1997.